

Problem

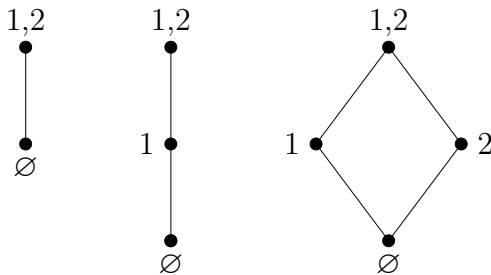
Find all knowledge spaces on a set Q with 2 and 3 elements.

Solution.

It is useful to organize the search systematically by the cardinality of the basis. Denote the basis by \mathcal{B} .

Two elements in Q

For $|Q| = 2$ the basis has 1 or 2 elements. There is one space with $|\mathcal{B}| = 1$ and two spaces with $|\mathcal{B}| = 2$. Here they are:

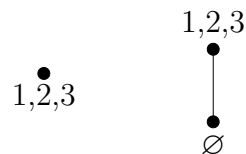


Thus, there are three different knowledge spaces; two of them are discriminative and well-graded and ordinal.

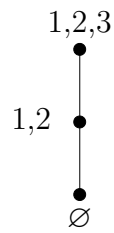
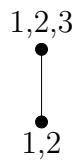
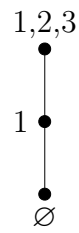
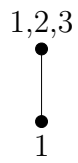
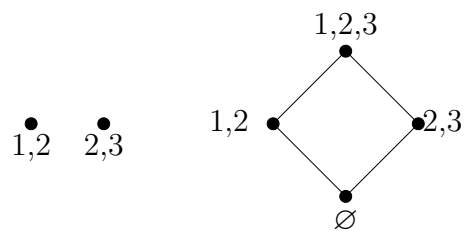
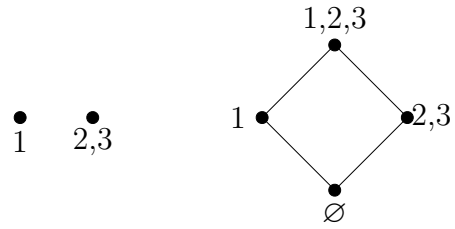
Three elements in Q

For $|Q| = 3$ the basis can have 1, 2, 3, or 4 element. In the following pictures we give the basis on the left side and the space on the right.

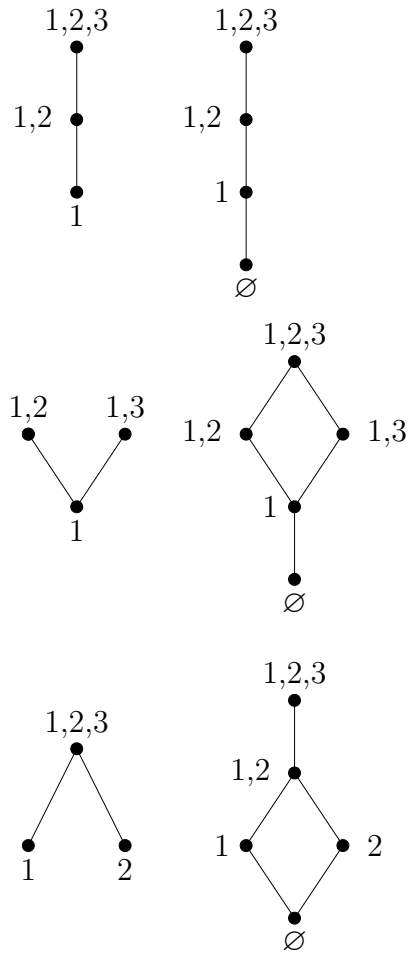
$|\mathcal{B}| = 1$



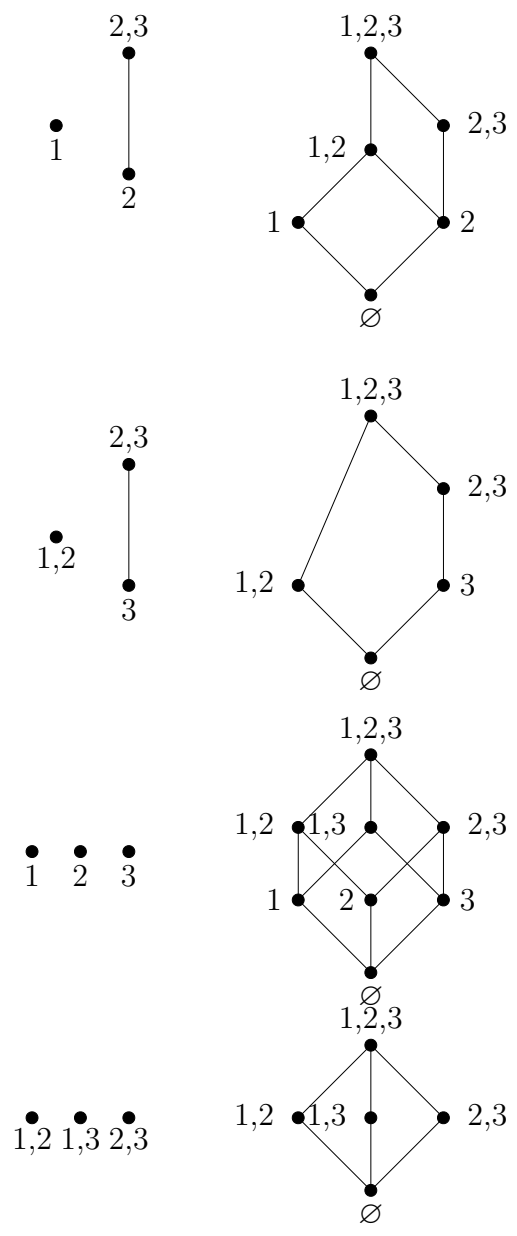
$$|\mathcal{B}| = 2$$



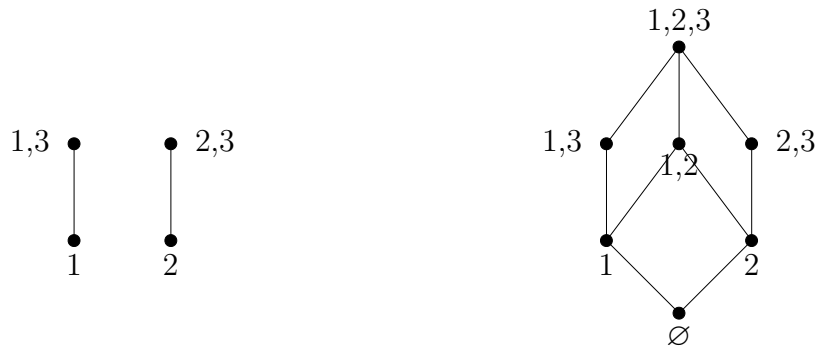
$$|\mathcal{B}| = 3$$



$|\mathcal{B}| = 3$ continued



$$|\mathcal{B}| = 4$$



Altogether, there are 13 knowledge spaces on a set Q of three elements; 9 are discriminative, and 6 are learning spaces, 8 are quasi ordinal, and 5 are ordinal.